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AIR FORCE OFFICE OF SCIENTIFIC RESEARCH  
GUGGENHEIM AERONAUTICAL LABORATORY  
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## ABSTRACT

A systematic analysis of Newtonian flow past an axi-symmetrical blunt body is developed by expanding various physical quantities in power series of  $\lambda = (\gamma-1)/(\gamma+1)$ , where  $\gamma$  stands for the ratio of specific heats of a gas.

Some general results such as stand-off distance, pressure distribution along the axis of symmetry are given, provided that the shock is detached and has finite curvature at its nose.

More extensive calculations are made for the flow past a flat-faced disc and power-law bodies.

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## I. INTRODUCTION

So far, the hypersonic flow theory of a perfect gas with a constant ratio of specific heats has been developed for two kinds of bodies such as very slender bodies as well as blunt-nosed bodies. While the former has been successfully treated by the method of small disturbances the latter case has been discussed a little later by using the further assumption that the adiabatic index of a gas is very close to unity. Such a flow is called now a "Newtonian" flow and much progress has been made already (Freeman<sup>(1)</sup> and Chester<sup>(2), (3)</sup> and the early paper of Busemann<sup>(4)</sup>).

Van Dyke<sup>(5)</sup> gave a concise review of the blunt body problem and developed a numerical method suited to a high speed computer.

Physically speaking, the difficulty of the flow with a detached shock wave comes from the fact that the location and the shape of a shock wave cannot be predicted in advance and the whole field is highly vortical due to the non-uniformity of strength of a curved shock wave. Therefore, many writers have tried to solve the inverse problem, in which the body shape is to be found for various assumed shock waves. But the shape of shock is not so sensitive to the change of body contour and the direct method, in which the body is given, is preferable.

In this paper, we assume a perfect gas with a constant ratio of specific heats and neglect various effects of a real gas at high temperature such as dissociation, recombination and ionization.

The density ratio  $\lambda$  across the normal portion of shock for infinite Mach number is related to the ratio of specific heats  $\gamma$ , and is the most important parameter for hypersonic flow.

But, the introduction of the second parameter  $B$  defined by  $1/(M_\infty^2 \lambda)$  extends the applicability of the theory for the case of infinite Mach number to that of high but finite Mach number.

The basic assumption for the deviation of the shock inclination from the right angle is of the order of  $\sqrt{\lambda}$  leads us to a consistent system of expansion for various physical quantities and we can develop a systematic procedure of successive approximation.

In Section 2, the fundamental equations and the boundary conditions are discussed. It will be seen that the dominant terms of both pressure coefficient and density ratio are constants but this does not mean the first

approximation corresponds to the constant density solution, because the second terms for these quantities are included in our first order equations. Moreover, the flow field is highly vortical and the system of equations for dominant terms has two real characteristics.

As is well known, characteristic coordinates are useful in a treatment of partial differential equations of hyperbolic type and we shall introduce such independent variables throughout this work.

As far as the first approximation is concerned, we have no mathematical difficulty and we can obtain some general results independent of the individual body shape for the stand-off distance and the pressure and velocity distribution along the axis of symmetry, provided that the shock wave is detached and has a finite curvature at its nose (Section 3).

In the following two sections, Sections 4 and 5, we shall treat in detail the two special cases--flat-faced disc and power-order bodies. The latter case may include the flow past a cone.

In the case of a cone, we get either flow patterns with an attached strong or weak shock wave or that with a detached shock wave, depending on the vertex angle of a cone, as is expected. Qualitative discussions by the aid of trajectories, however, predict more possible shock configurations in each case, which have a close similarity to the flow past an infinite wedge shown by the author a few years ago. (6), (7)

Recently, Serbin<sup>(8)</sup> and Freeman<sup>(9)</sup> attacked the blunt body problem by similar methods, independently. The scheme of expansion used by them is similar to the one presented here although some details are different.

## 2. FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

Assuming that the gas is perfect and has a constant ratio of specific heats,  $\gamma$ , we shall define two parameters by the formulae:

$$\lambda = \frac{\gamma-1}{\gamma+1}, \quad B = \frac{1}{\lambda M_\infty^2} \quad (2.1)$$

where  $M_\infty$  stands for the Mach number at infinity upstream. As will be seen in Eq. (2.5), these two parameters are related to the density jump through the normal portion of a shock.

In the following analysis, not only  $M_\infty^2$  but also  $1/\lambda$  are taken to be very large, while the parameter  $B$  is kept finite. The theory based on these assumptions is now called the 'hypersonic Newtonian flow theory.'

Now, we shall make the further assumption that the shock wave is almost normal to the oncoming flow in such a way that the deviation of the shock angle from  $\pi/2$  is of the order of  $\sqrt{\lambda}$ . The justification for this procedure is a consistent set of approximation equations.

Consistent with this assumption, we shall expand various physical quantities in power series of  $\lambda$ , whose dominant terms may or may not be of the fractional order.

Taking the Cartesian coordinates  $(x, r)$  with the origin at the nose of a body and  $x$ -axis coincident with the axis of symmetry (see Fig. 1), and letting  $P, \rho, Q, Q_x, Q_r, \Theta$  be pressure, density, magnitude of velocity, velocity components in  $(x, r)$  direction and the shock inclination, respectively, we shall expand these quantities in the following terms:

$$\begin{aligned} \Theta &= \pi/2 - \sqrt{\lambda}(\theta_1 + \lambda\theta_2 + \dots) \\ \frac{P - P_\infty}{\rho_\infty Q_\infty^2} &= 1 + \lambda p_1(x^*, r) + \lambda^2 p_2(x^*, r) + \dots \\ \frac{\rho}{\rho_\infty} &= \frac{1}{\lambda} \left( \frac{1}{1+B} + \lambda \sigma_1(x^*, r) + \lambda^2 \sigma_2(x^*, r) + \dots \right) \\ \frac{Q}{Q_\infty} &= \lambda(u_1(x^*, r) + \lambda u_2(x^*, r) + \dots) \\ \frac{Q_r}{Q_\infty} &= \sqrt{\lambda}(v_1(x^*, r) + \lambda v_2(x^*, r) + \dots) \end{aligned} \quad (2.2)$$

where the suffix  $\infty$  refers to the undisturbed flow at infinity upstream and  $x^*$  is the reduced coordinate defined by the formula

$$x^* = x/\sqrt{\lambda} \quad (2.3)$$

Lower case quantities are non-dimensional and their suffixes show the order of approximation. The dominant terms in pressure coefficient and density ratio are constants, which will be seen to be a direct consequence of our analysis.

It is to be noted that the Mach number,  $M$ , at any point is given by

$$M^2 = \frac{Q_x^2 + Q_r^2}{\gamma P/\rho} = \frac{v_1^2}{1+B} \times (1 + \dots) \quad (2.4)$$

Next, we shall write down the well-known shock relations in series of  $\lambda$ :

$$\begin{aligned} \frac{Q_x}{Q_\infty} &= \lambda(1 + B + \theta_1^2) + \dots \\ \frac{Q_r}{Q_\infty} &= \sqrt{\lambda}(\theta_1 + \dots) \\ \frac{P - P_\infty}{\rho_\infty U_\infty^2} &= 1 - \lambda(1 + B + \theta_1^2) + \dots \\ \frac{\rho}{\rho_\infty} &= \frac{1}{\lambda(1+B)} (1 + \lambda \frac{B}{1+B} (1 - \theta_1^2) + \dots) \end{aligned} \quad (2.5)$$

which must hold on the shock wave. It should be borne in mind that  $\theta_1, \theta_2, \dots$  are not given a priori but are to be determined later.

The fundamental equations governing flow field are given by

$$\begin{aligned} Q_x \frac{\partial Q_x}{\partial x} + Q_r \frac{\partial Q_x}{\partial r} &= - \frac{1}{\rho} \frac{\partial P}{\partial x} \\ Q_x \frac{\partial Q_r}{\partial x} + Q_r \frac{\partial Q_r}{\partial r} &= - \frac{1}{\rho} \frac{\partial P}{\partial r} \\ \frac{\partial}{\partial x} (\rho Q_x r) + \frac{\partial}{\partial r} (\rho Q_r r) &= 0 \\ Q_x \frac{\partial}{\partial x} (P/\rho^\gamma) + Q_r \frac{\partial}{\partial r} (P/\rho^\gamma) &= 0 \end{aligned} \quad (2.6)$$

The first two are the momentum equations in  $(x, r)$  directions, re-

spectively, the third shows the conservation of mass and the fourth refers to the entropy conservation along each streamline.

From the third equation, we can define the stream function,  $\Psi$ , by

$$\frac{\partial \Psi}{\partial r} = \rho Q_x r, \quad \frac{\partial \Psi}{\partial x} = -\rho Q_r r \quad (2.7)$$

But, we shall introduce the reduced stream function,  $\psi$ , by the relation:

$$\Psi = \frac{1}{1+B} \rho_\infty Q_\infty \psi \quad (2.8)$$

so that we may have the simplified relation in place of Eq. (2.7)

$$\frac{\partial \psi}{\partial r} = u_1 r, \quad \frac{\partial \psi}{\partial x} = -v_1 r \quad (2.9)$$

The equations of higher order are easily obtained in a similar way.

Substituting Eq. (2.2) into Eq. (2.6) and retaining only the lowest order terms, we have

$$\begin{aligned} (u_1 \frac{\partial}{\partial x} + v_1 \frac{\partial}{\partial r}) u_1 &= -(1+B) \frac{\partial p_1}{\partial x} \\ (u_1 \frac{\partial}{\partial x} + v_1 \frac{\partial}{\partial r}) v_1 &= 0 \\ \frac{\partial}{\partial x} (u_1 r) + \frac{\partial}{\partial r} (v_1 r) &= 0 \\ (u_1 \frac{\partial}{\partial x} + v_1 \frac{\partial}{\partial r}) (p_1 - (1+B) \sigma_1) &= 0 \end{aligned} \quad (2.10)$$

The first and fourth equations show that the dominant terms of pressure and density do not change along each streamline and according to the shock conditions, Eq. (2.5), they must be constants throughout the field, whose values are used already in Eq. (2.2).

It is easily shown that Eqs. (2.10) have two real characteristics:

$$r = \text{const.} \quad \text{and} \quad \psi = \text{const.}$$

which are conveniently used as independent variables. However, we define the new variable  $s$  instead of  $\psi$  by the relation:

$$\psi = \frac{1}{2} (1+B) s^2 \quad (2.11)$$

and  $r$  and  $s$  are taken as new independent variables. The physical meaning



of  $s$  is the radial distance of the point on a shock wave, from which the streamline going through the point  $(r, s)$  starts (see Fig. 2).

The introduction of  $s$  has an advantage that the shock can be denoted by the simple relation  $r = s$  and the fundamental equations (2.10) are reduced to the following forms:

$$\begin{aligned}\frac{\partial u_1}{\partial r} &= \frac{r}{s} \frac{\partial p_1}{\partial s} \\ \frac{\partial v_1}{\partial r} &= 0 \\ \frac{\partial}{\partial s} \left( \frac{u_1}{v_1} \right) &= \frac{1+B}{r^2} \frac{s}{v_1} \\ \frac{\partial}{\partial r} (p_1 - (1+B)\sigma_1) &= 0\end{aligned}\tag{2.12}$$

The boundary conditions, to which the solutions of Eqs. (2.12) are subject, are written in the form

$$\begin{aligned}u_1 &= 1 + B + \theta_1^2 \\ v_1 &= \theta_1 \\ p_1 &= -(1 + B + \theta_1^2) \\ \sigma_1 &= \frac{B}{(1+B)^2} (1 - \theta_1^2)\end{aligned}\tag{2.13}$$

on  $r = s$ , and

$$\frac{dx^*}{dr} = \frac{u_1}{v_1}\tag{2.14}$$

on  $s = 0$ .

The latter corresponds to the condition that the streamline must be tangent to the surface of the body.

Finally it is convenient for the first approximation problem to eliminate the parameter  $1 + B$  by the transformations

$$u_1(x^*, r) = (1+B)u(\tilde{x}, r)$$

$$v_1 = \sqrt{1+B} v$$

$$p_1 = (1+B)p$$

$$x^* = \sqrt{1+B} \tilde{x}$$

$$\theta_1 = \sqrt{1+B} \theta$$

(2.15)

The basic equations and boundary conditions thus read

$$\frac{\partial u}{\partial r} = \frac{r}{s} \frac{\partial p}{\partial s}$$

$$\frac{\partial v}{\partial r} = 0$$

$$\frac{\partial}{\partial s} \left( \frac{u}{v} \right) = \frac{s}{r^2} \frac{1}{v}$$

$$\frac{\partial}{\partial r} (p - \sigma_1) = 0$$

(2.16)

at the shock

$$u = 1 + \theta^2$$

$$v = \theta$$

$$p = -(1 + \theta^2)$$

$$\sigma_1 = \frac{B}{(1+B)^2} (1 - (1+B)\theta^2)$$

(2.17)

and at the body

$$\frac{d\tilde{x}}{dr} = \frac{u}{v}$$

(2.18)

### 3. THE FIRST APPROXIMATION--GENERAL RESULTS

The solutions for Eqs. (2.12) subject to the shock conditions (2.13) are easily obtained.

$$\begin{aligned} u(s, r) &= \theta(s) \left[ \frac{1}{r^2} \int_r^s \frac{t dt}{\theta(t)} + \frac{1 + \theta^2(r)}{\theta(r)} \right] \\ u(s, r) &= \theta(s) \\ p(s, r) &= -1 - \theta^2(r) + \frac{1}{r} \int_r^s t \frac{\partial u(t, r)}{\partial r} dt \end{aligned} \quad (3.1)$$

$$\sigma_1(s, r) = \frac{B}{(1+B)^2} + \frac{\theta^2(s)}{1+B} - \theta^2(r) + \frac{1}{r} \int_r^s t \frac{\partial u(t, r)}{\partial r} dt$$

To determine the unknown function  $\theta$ , we shall consider the boundary condition on the body. Substituting Eqs. (3.1) into Eq. (2.18), we have

$$\left[ \frac{d\tilde{x}}{dr} \right]_{\text{Body}} = -\frac{1}{r^2} \int_0^r \frac{t dt}{\theta(t)} + \frac{1 + \theta^2(r)}{\theta(r)} \quad (3.2)$$

on which the discussions of the following three sections are based.

When the body shape is given, Eq. (3.2) appears as an integral equation for  $\theta$ , while it is a differential equation for determining the body shape in case of the given shock shape.

However, we can get some general formulae independent of the body shape, provided that the curvature of the shock wave at its nose is finite.

In accordance with this assumption, we shall expand  $\theta(s)$  in the Taylor series:

$$\theta(s) = s\theta'(0) + O(s^2) \quad (3.3)$$

where  $\theta'(0)$  is related to the radius of curvature of a shock at its nose by the relation

$$\theta'(0) = \frac{1}{\sqrt{\lambda} R_s}, \quad R_s = \text{radius of curvature of shock at } r = 0 \quad (3.4)$$

From Eqs. (2.9) and (2.11), the reduced coordinates  $\tilde{x}$  are given by

$$\tilde{x}(s, r) - \tilde{x}(r, r) = \frac{1}{r} \int_s^r (t/\theta(t)) dt \quad (3.5)$$

Eq. (3.5) is considered as the equation of a streamline referred to the shock wave for the fixed value of  $s$ , but it is also used to estimate the stand-off distance of a detached shock wave (if any) from a body by taking ( $s=0$ ,  $r \rightarrow 0$ ).

Let  $\delta$  be the stand-off distance along the axis of symmetry. We obtain from Eqs. (3.5) and (3.3)

$$\frac{\delta}{\sqrt{\lambda}} = \lim_{r \rightarrow 0} (\tilde{x}(0, r) - \tilde{x}(r, r)) = \frac{1}{\theta'(0)}$$

This is rewritten in more convenient form by using Eq. (3.4)

$$\frac{\delta}{R_s} = (1+B)\lambda \quad (3.6)$$

that is, shock separation is  $O(\lambda)$  times shock radius.

The velocity, the pressure and the density distributions along the axis of symmetry are obtained in a similar way. In these cases, we have to let  $r$  approach zero for a fixed value of  $s/r$ , say  $k$ . Evaluating  $u$ ,  $p$ ,  $\sigma$  and  $\tilde{x}$  in terms of  $k$  and then eliminating the parameter  $k$ , we can obtain the following general formulae:

$$\begin{aligned} u(x) &= (x/\delta)^2 \\ p(x) &= -\frac{1}{2} (1 + (x/\delta)^4) \\ \sigma_1(x) &= \frac{1}{2} \left( 1 - \left( \frac{x}{\delta} \right)^4 \right) + \frac{B}{(1+B)^2} \end{aligned} \quad (3.7)$$

Here, the distance  $x$  is measured from the nose of a body.

#### 4. HYPERSONIC FLOW TOWARD A FLAT-FACED DISC

As a first example, we shall consider the hypersonic flow toward a flat-faced disc with radius  $r_0$  placed normal to the oncoming uniform flow. In this case, the boundary condition on the body is expressed in the form:

$$\left( \frac{dx}{dr} \right)_{s=0} = 0 \quad \text{for } r \leq r_0 \quad (4.1)$$

Then, the integral equation for  $\theta$  from Eq. (3.2) is

$$\frac{1 + \theta^2(r)}{\theta(r)} = \frac{1}{r^2} \int_0^r \frac{t dt}{\theta(t)}$$

Multiplying by  $r^2$  and differentiating with respect to  $r$ , we can convert this to the differential equation:

$$\frac{d\theta}{dr} = \frac{\theta}{r} \frac{1+2\theta^2}{1-\theta^2} \quad (4.2)$$

Eq. (4.2) is easily integrated to give the solution:

$$r = A \frac{\theta}{(1+2\theta^2)^{3/4}}, \quad A = \text{const.} \quad (4.3)$$

In order to interpret the constant  $A$ , we proceed as follows.

Take  $r_0$  as a reference length (see Fig. 3). For this purpose, trajectories of Eq. (4.2) are shown in Fig. 4. It is easily proved that the origin in  $(r, \theta)$ -plane behaves as a nodal point and the line  $\theta = 1$ , which corresponds to the sonic line in the first approximation (as is seen from Eqs. (2.4) and (3.1)), is the isocline for vertical trajectories. Since the sign of  $d\theta/dr$  changes across this line, all integral curves starting from the origin must turn back towards the  $\theta$ -axis at some critical point ( $r = r_0$ ) on the sonic line. In the physical plane, this appears as a cusp on the shock wave. It is reasonable to identify  $r_0$  with the radius of the disc, because any co-axial cylinder,  $r = \text{const.}$ , is one of the characteristics of Eq. (2.10). Any singularity of the equation must propagate along the characteristic. By making use of this convention, we can rewrite Eq. (4.3) in terms of the Mach number  $M$  as

$$\frac{r}{r_0} = \frac{3^{3/4} M}{(1+2M^2)^{3/4}} \quad (4.4)$$

with

$$M^2 = \theta^2$$

The appearance of a singular point on a shock restricts the validity of this solution and if we proceed to higher approximation, this drawback becomes more serious.

Now that  $\theta$  is determined, velocity components, pressure and density are derived by straightforward calculations in the forms:

$$\begin{aligned} u(s, r) &= (s/r)^2 (\theta^2(s) + 1) \\ p(s, r) &= -(1 + \theta^2(r)) + \frac{1}{16} (r_0/r)^4 (\varphi(s) - \varphi(r)) - \frac{3}{4} (1 - s^4/r^4) \end{aligned} \quad (4.5)$$

where

$$\varphi(s) = 27 \log(1 + 2\theta^2(s)) + (s/r_0)^4 \left( 13 \frac{1}{\theta^2(s)} + \frac{5}{2} \frac{1}{\theta^4(s)} \right)$$

The pressure distribution according to (4.6) is plotted in Fig. 5a. Though Eqs. (4.6) are of rather complicated form, they are free from any singularity and, especially, the pressure distribution on the surface of a body can be expanded for small values of  $r$  in the form:

$$p_1(0, r) = -\frac{1}{2} - \frac{2}{9} \frac{1}{\sqrt{3}} \left( \frac{r}{r_0} \right)^2 + O(r/r_0)^4 \quad (4.6)$$

To evaluate a shock shape, it is better to use Eq. (2.10) in the Cartesian coordinates  $(x, r)$ . It follows from the second and third equations, that

$$\frac{\partial}{\partial x} \left( \frac{u}{v} \right) = -\frac{1}{r}$$

Integrating this equation with respect to  $x^*$  for fixed  $r$  from a shock to a body, and using the shock conditions, Eq. (2.13), we have

$$\frac{\tilde{x}}{r} = \frac{1}{\theta} (\theta^2 + 1)$$

where  $\tilde{x}$  denotes the distance between a shock and a body measured along the line  $r = \text{const.}$

Together with Eq. (4.5), we can express the equation of the shock in the parametric form:

$$\left( \frac{\tilde{x}}{r_0} \right)^4 = \frac{27}{(1+2M^2)^3} (1+M^2)^4, \quad \left( \frac{r}{r_0} \right)^4 = \frac{27 M^2}{(1+2M^2)^3} \quad (4.8)$$

## 5. HYPERSONIC FLOW PAST A POWER-ORDER BODY

In this section, we discuss the flow past a power-order body, whose contour in the meridian plane is given by

$$x^* = cr^p \sqrt{1+B} \quad \text{or} \quad \tilde{x} = cr^p \quad (5.1)$$

where  $c$  and  $p$  are two positive constants.

Substituting this relation into Eq. (3.2) and making some simple calculations, we have the differential equation for  $\theta$  in the form

$$\frac{d\theta}{dr} = \frac{\theta(cp(p+1)r^{p-1}\theta - 1 - 2\theta^2)}{r(\theta^2 - 1)} \quad (5.2)$$

Now, we shall distinguish three cases depending on the value of  $p$ .

(1)  $p > 1$ .

This case includes the flow past a paraboloid of revolution ( $p = 2$ ); the qualitative nature of the flow pattern for all other cases is similar to this typical one.

(2)  $p < 1$ .

The power order bodies classified in this case have a cusp at the nose and our basic assumptions for the order of magnitude are obviously violated in a region near the nose. Thus, we shall exclude this case from our consideration.

(3)  $p = 1$ .

This case corresponds to the flow past a cone and Eq. (5.2) is reduced to the form:

$$\frac{d\theta}{dr} = - \frac{\theta(2\theta^2 - 2c\theta + 1)}{r(\theta^2 - 1)} \quad (5.3)$$

A quadratic expression of  $\theta$  is a factor of the numerator of Eq. (5.3) and three different types of solutions, which should be treated separately, are obtained depending on the roots of this quadratic equation.

(i) The case for  $c^2 > 2$

In this case Eq. (5.3) is rewritten in the form:

$$\frac{d\theta}{dr} = -2 \frac{\theta(\theta - \theta^{(1)})(\theta - \theta^{(2)})}{(\theta - 1)(\theta + 1)} \quad (5.4)$$

where

$$\theta^{(1), (2)} = \frac{c \pm \sqrt{c^2 - 2}}{2} \quad (\theta^{(1)} \geq \theta^{(2)})$$

The behavior of  $\theta^i$  ( $i = 1, 2$ ) against  $c$  is shown in Fig. 5.

Since the sonic point corresponds to  $\theta = 1$ , we shall further distinguish three more cases, which are given in Table I.

TABLE I

Case	(a)	(b)	(c)
$c$	$c > \frac{3}{2}$	$c = \frac{3}{2}$	$\frac{3}{2} > c > \sqrt{2}$
$\theta^{(1)}$	supersonic	sonic	subsonic
$\theta^{(2)}$		subsonic	

Though it is not difficult to get the complete solution of Eq. (5.4), it is enough to study the behavior of trajectories of Eq. (5.4) for predicting the flow pattern.

First of all, we notice that the origin in  $(r, \theta)$ -plane is the nodal point of Eq. (5.4), from which the integral curves may start, and the singular point at  $r = 0$ ,  $\theta = \theta^{(2)}$  is a saddle point in all three cases; however the point  $(0, \theta^{(1)})$  may be a nodal point, a regular point, or a saddle point corresponding to the cases (c), (b) or (a) respectively. Furthermore, it is obvious that the four straight lines

$$\theta = \theta^{(1)}, \theta^{(2)}, 0 \text{ and } r = 0 \quad (5.5)$$

are integral curves of Eq. (5.4) except for the line  $\theta = \theta^{(1)}$  for the case (b).

In Fig. 6(a), (b), (c) are shown the typical integral curves of Eq. (5.4) for three cases, respectively. The plus or minus sign in these figures refers to the sign of  $d\theta/dr$  in each region.

In order to have a physical interpretation, only the integral curves starting from the point on the line  $r = 0$  are permissible and they are shown by thick lines.

The line  $\theta = 0$  ( $r$ -axis) corresponds to the trivial case, where the shock is normal and straight and retreats to infinity upstream.

Two straight lines (I)  $\theta = \theta^{(1)}$  and (II)  $\theta = \theta^{(2)}$  correspond to the weak and the strong straight (conical) shock waves, respectively, and both of them are attached to the cone, as is easily checked by Eq. (3.5).

Besides these two shock configurations, there are a few other possi-



bilities, all of which give curved shock waves.

The curve marked (III) in Fig. 6 starts from the origin in a certain direction, which is related to the curvature of the shock (arbitrary because of the non-existence of any reference length) and finally approaches the line (II). Mapping back to the physical plane, we find a detached shock wave starting as a normal shock with a finite curvature at its nose and tending to a strong shock as we move along it. The stand-off distance referred to the radius of curvature is given by the general formula, Eq. (3.6).

In case (c), two sets of integral curves start from the nodal point at  $(r = 0, \theta = \theta^{(1)})$  and one of them marked (IV) in Fig. 6(c) comes down to tend to the line (II), while the other labelled (V) goes up and right until a certain critical point on the sonic line, then turns back to the left and finally tends to the  $\theta$ -axis. The former corresponds to the shock starting as a weak shock at the vertex of a cone and tends to a strong shock gradually but the latter is transformed to the shock with a cusp at the sonic point just as in the case of the previous section.

The curvature of these shock waves at their nose may be zero or infinite depending on the value of  $c$ , and there is one special value,  $c \doteq 1.4506$ , for which the curvature remains finite. This situation just corresponds to the 'Crocco state' of the two-dimensional flow past a wedge.

All the possible shock configurations are schematically shown in Fig. 7.

In some earlier papers<sup>(6), (7)</sup>, the author pointed out theoretically the existence of curved shock waves due to an infinite wedge, though there is no reference length in this problem. It is interesting to note that the situation is very similar to the present cases, especially to the case (c).

(ii) The case for  $c^2 = 2$

The equation to determine  $\theta$ , (5.3), is brought into the form:

$$\frac{d\theta}{dr} = -2 \frac{\theta}{r} \frac{(\theta - \theta^*)^2}{(\theta - 1)(\theta + 1)} \quad (5.6)$$

with

$$\theta^* = 1/\sqrt{2}$$

It follows from Eq. (5.6) that  $d\theta/dr$  changes its sign only across the sonic line and vanishes on the line  $\theta = \theta^*$ .

The behavior of typical integral curves are shown in Fig. 8. This, of course, is the limiting case of (ic), when (I) and (II) coincide each other and (IV) vanishes. Three possible shock configurations in the physical plane

are shown in Fig. 9.

(iii) The case for  $c^2 < 2$

The last case is rather simple, because the numerator of Eq. (5.3) remains positive for any positive value of  $\theta$ .

A trajectory of Eq. (5.3) in the  $(r, \theta)$ -plane as well as its mapping to the physical plane are drawn in Fig. 10.

There is only one possible flow pattern, which is completely similar to the case of a flat-faced disc.

## 6. INVERSE PROBLEM FOR A POWER - ORDER SHOCK WAVE

So far, the 'direct' problem for a given body has been exclusively attacked in this paper, but the conventional 'inverse' method for an assumed shock wave can be studied more easily.

Here, we shall assume the shock shape is given by

$$\tilde{x} = mr^q \quad (q \geq 1) \quad (6.1)$$

Then, by definition,  $\theta$  is

$$\theta = \frac{d\tilde{x}}{dr} = mqr^{q-1} \quad (6.2)$$

Using Eq. (6.2) into Eq. (3.2) and integrating with respect to  $r$ , we have

$$\tilde{x}_{\text{body}} = \frac{1}{mq(3-q)} r^{2-q} + mr^q + \tilde{\delta} \quad (6.3)$$

where  $\tilde{\delta}$  is the integration constant equal to the stand-off distance of the shock when the origin is taken at the location of a shock intersection with the axis of symmetry.

In order to have a closed body, the value of  $q$  should be subject to the conditions, together with (6.1),

$$2 \geq q \geq 1 \quad (6.4)$$

Thus, for smaller values of  $r$ , the dominant term in Eq. (6.3) is  $O(r^{2-q})$  and due to the condition (6.4) the body has a cusp at its nose except the two limit cases:  $q = 1$  or  $2$ . Although bodies with such shapes were excluded from our consideration in the previous section, the formal correspondence between this case and case (2) in Section 5 can be shown easily.

For the two exceptional cases, Eq. (6.3) gives

$$\begin{aligned} \tilde{x}_{\text{body}} &= \left( \frac{1}{2m} + m \right) r & \text{for } q = 1 \\ \tilde{x}_{\text{body}} &= \frac{1}{2m} + mr^2 & \text{for } q = 2 \end{aligned} \quad (6.5)$$

Here  $\tilde{\delta}$  is omitted because not only the conical shock is attached to the conical body but also the first term of the second formula gives the exact result for stand-off distance.

It follows from the first equation of (6.5) that one conical shock always corresponds to one and only one conical body, but as seen in the previous section one conical body enables us to have various possible shock configurations. This is shown clearly in Fig. 5, where the 'c' is one valued function of  $\theta$  but not vice versa.

The second of Eq. (6.5) leads us to the deduction that the paraboloidal shock corresponds to the body of the exactly same shape as long as the first order approximation is concerned.

At first sight, this statement seems to be contradictory to the prediction made in Section 5. However, this apparent contradiction is remedied by a careful discussion in which an additional term is used to express the shock shape.

Consider Eq. (6.1) replaced by an alternative formula

$$\tilde{x} = mr^2(1 + nr^{2\epsilon}), \quad (\epsilon > 0) \quad (6.6)$$

Proceeding as before, we get from Eq. (3.2) together with Eq. (6.6)

$$\frac{d\tilde{x}}{dr} = -\frac{1}{r^2} \int_0^r \frac{dt}{2m(1+(1+\epsilon)nt^{2\epsilon})} + \frac{1}{2mr(1+(1+\epsilon)nr^{2\epsilon})} + 2mr(1+(1+\epsilon)nr^{2\epsilon})$$

Since we are interested in the region near the axis of symmetry, we can expand the denominator for small values of  $r$  and after some simple calculations we obtain the expression for the body shape in the form

$$\tilde{x}_{\text{body}} = -\frac{1}{2m} \frac{1+\epsilon}{1+2\epsilon} nr^{2\epsilon} + mr^2 + O(r^{2+2\epsilon}) \quad (6.7)$$

It is seen from this equation that  $n$  must be negative to obtain a physically realistic flow.

If  $\epsilon$  is less than one, the first term in Eq. (6.7) is dominant near the nose, though it corresponds to the correction term in Eq. (6.6). In other words, a small disturbance in the shock shape may produce a large influence on the body shape.

It is worthwhile to pay special attention to the case  $\epsilon \approx 1$ , in which Eqs. (6.6) and (6.7) reduce to the forms:

$$\begin{aligned} \tilde{x}_{\text{shock}} &= mr^2 + mn r^4 \\ \tilde{x}_{\text{body}} &= (m - \frac{n}{3m})r^2 + O(r^4) \end{aligned}$$

respectively.

Since  $n$  is negative, the curvature of the shock at its nose is less than that of the corresponding body.

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A systematic analysis of Newtonian flow past an axi-symmetrical blunt body is developed by expanding various physical quantities in power series of  $\lambda = (\gamma-1)/(\gamma+1)$ , where  $\gamma$  stands for the ratio of specific heats of a gas.

Some general results such as stand-off distance, pressure distribution along the axis of symmetry are given, provided that the shock is detached and has finite curvature at its nose.

More extensive calculations are made for the flow past a flat-faced disc and power-law bodies.

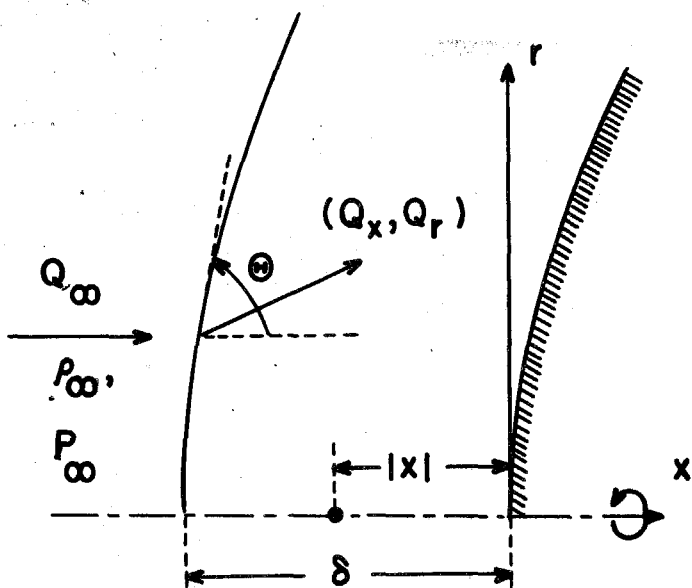


FIG. 1

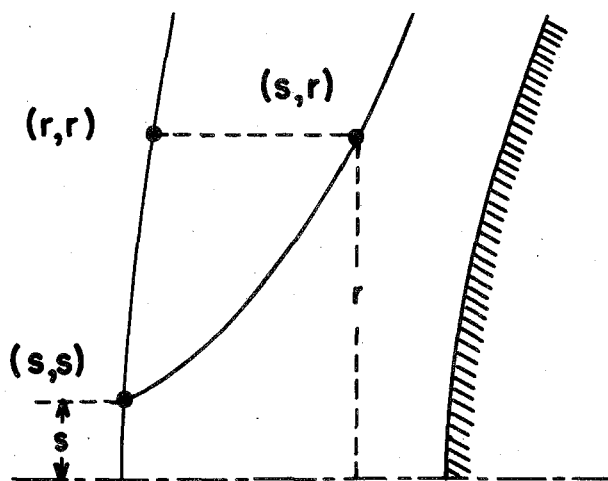


FIG. 2



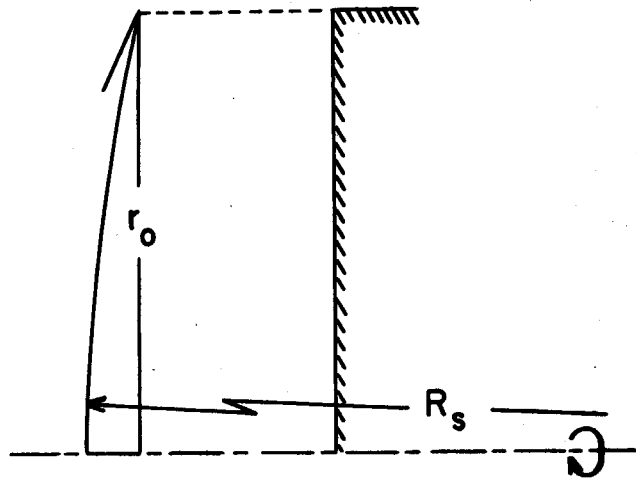


FIG. 3

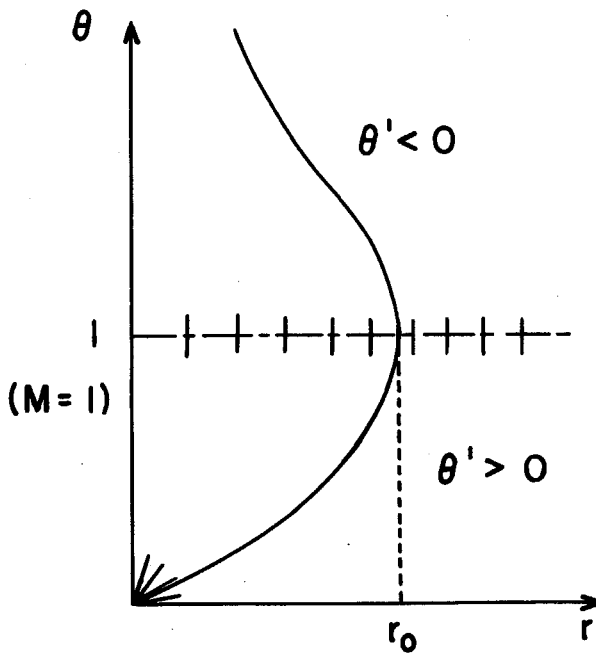


FIG. 4

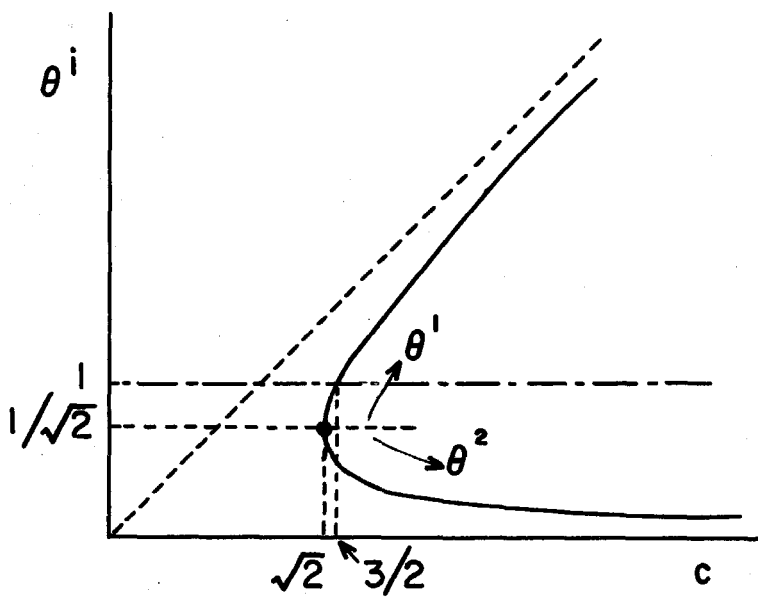


FIG. 5

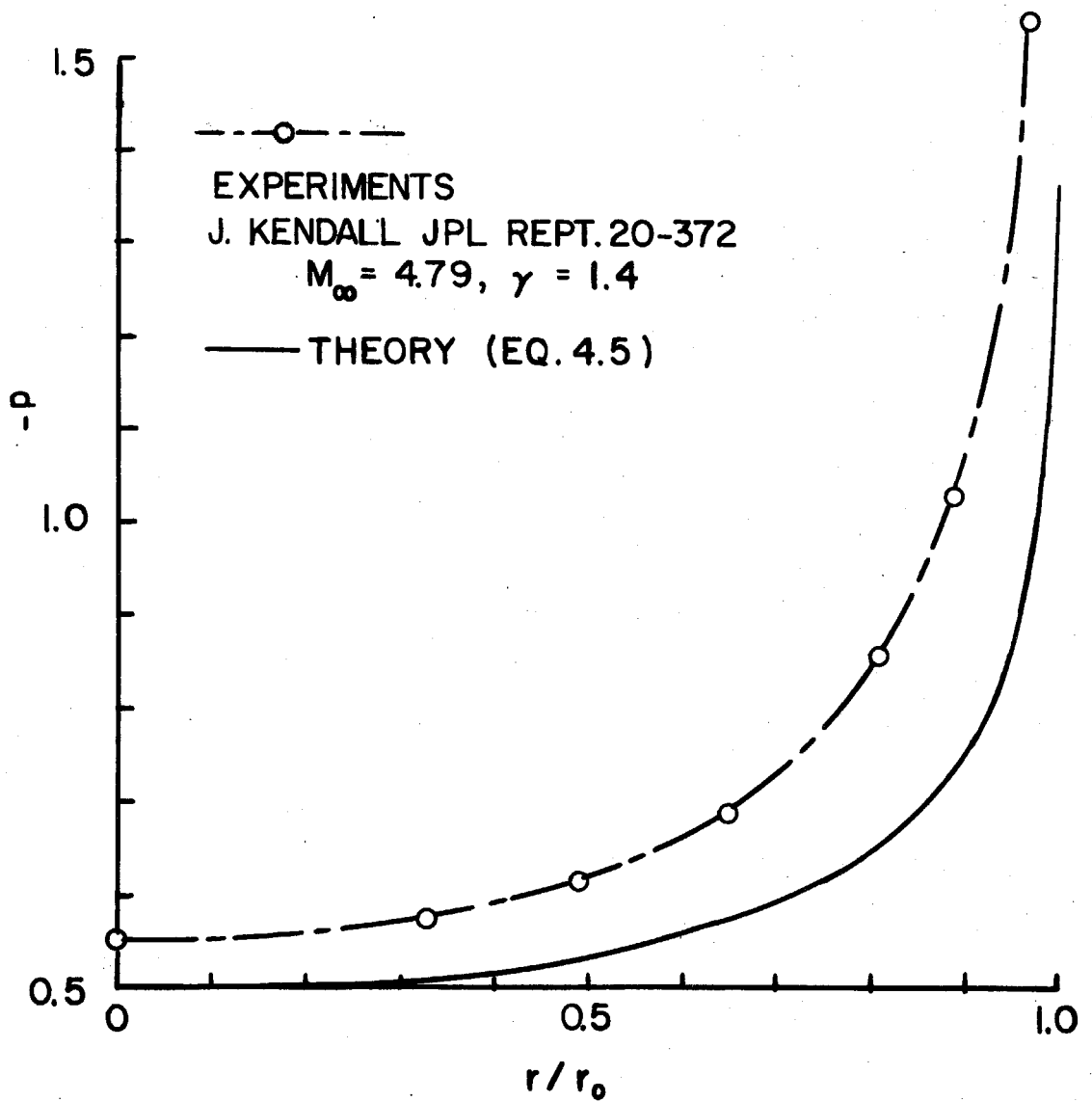


FIG. 5a  
PRESSURE DISTRIBUTION ON DISK

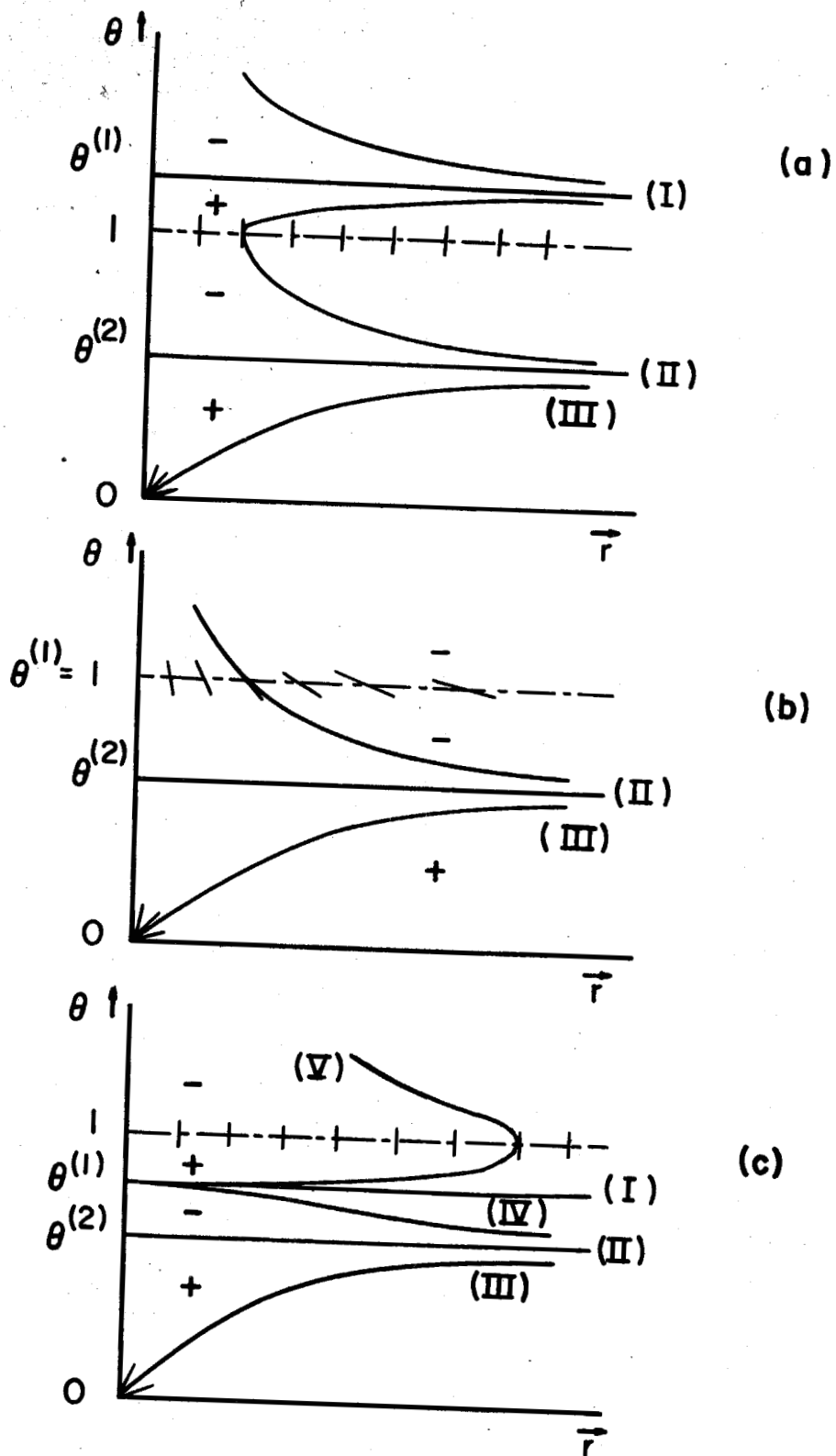


FIG. 6

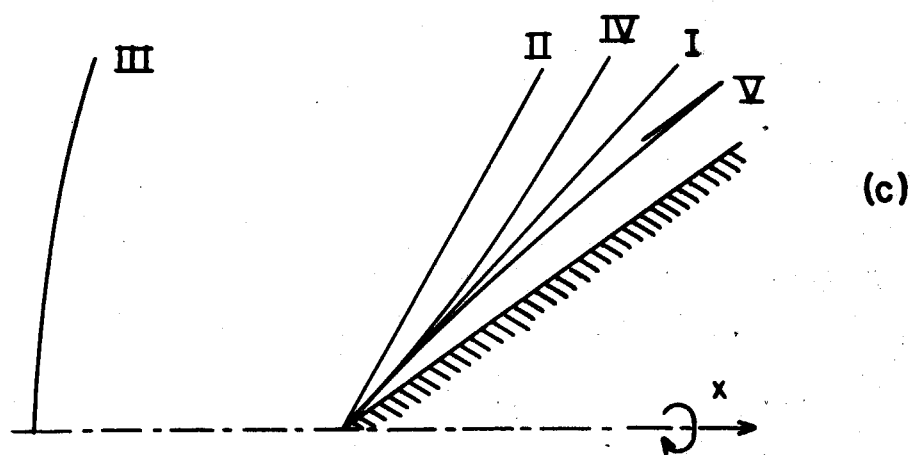
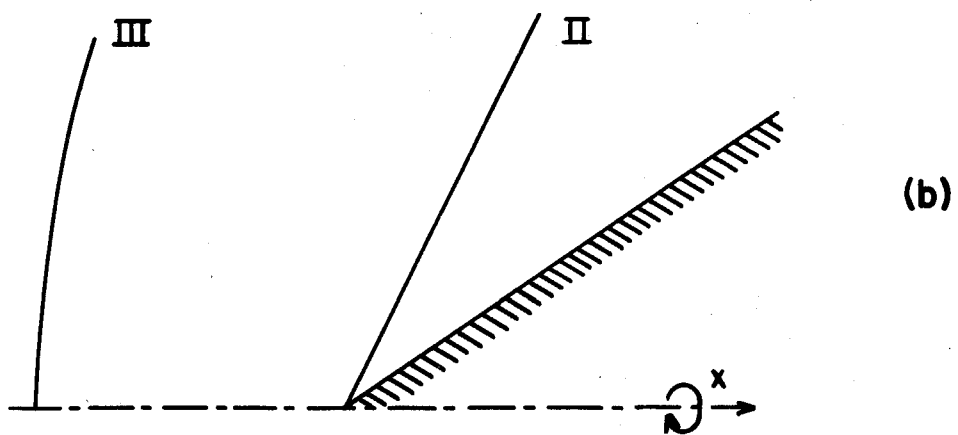
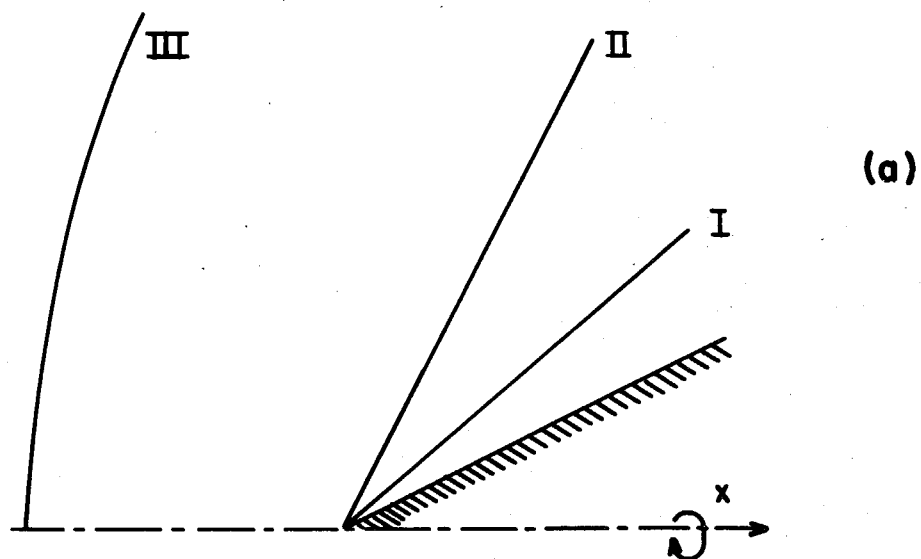


FIG. 7

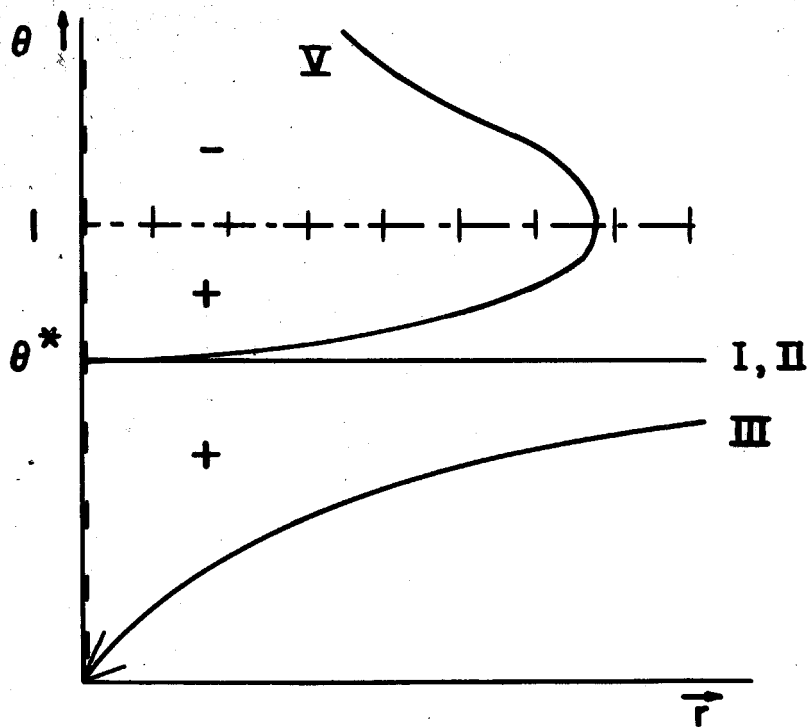


FIG. 8

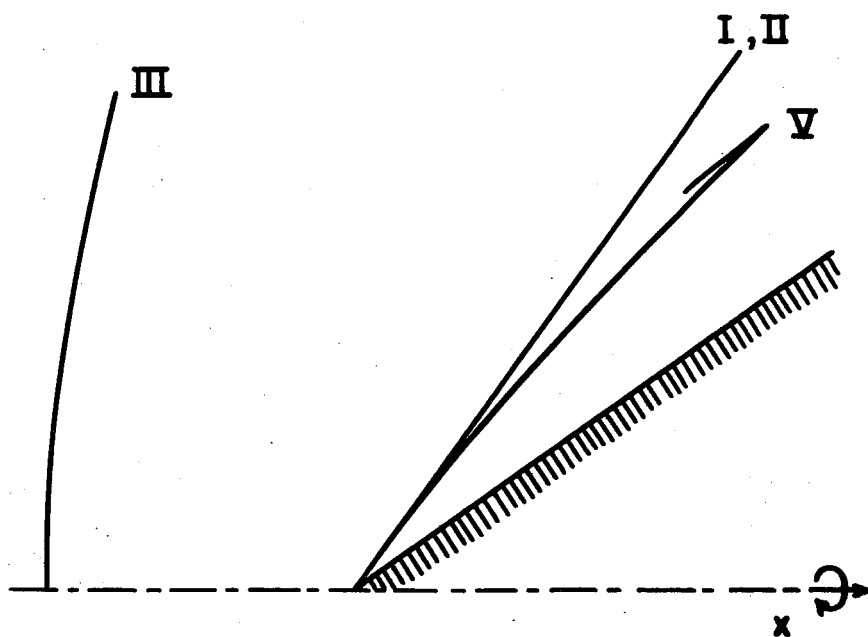


FIG. 9

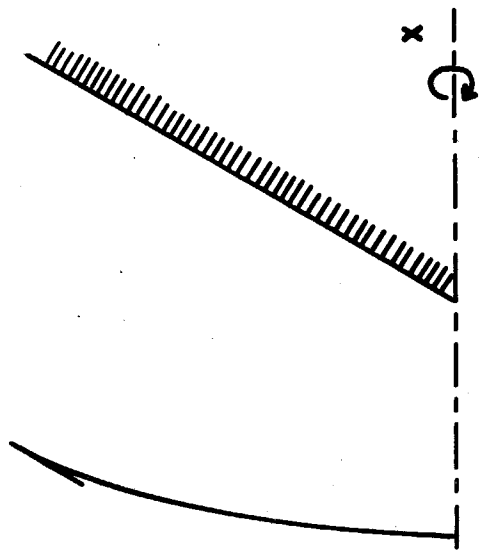
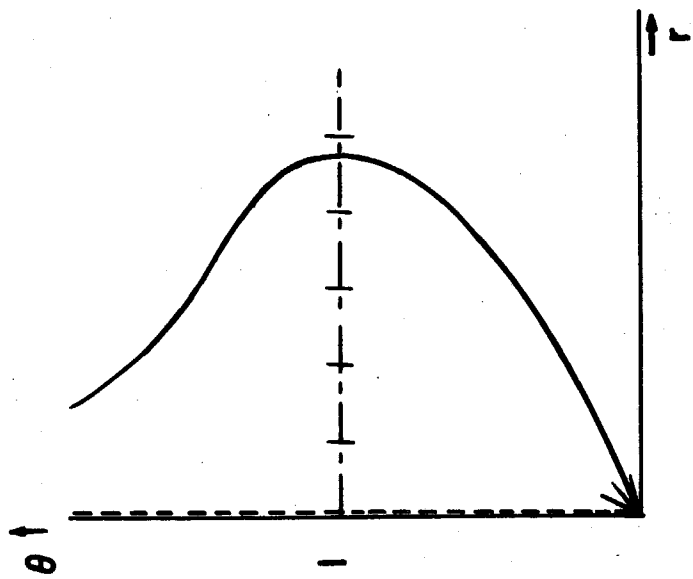


FIG. 10